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Monte Carlo simulations of increased/decreased scattering inclusions inside a turbid slab

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Abstract

We analyse the effect on scattered photons of anomalous optical inclusions in a turbid slab with otherwise uniform properties. Our motivation for doing so is that inclusions affect scattering contrast used to quantify optical properties found from transmitted light intensity measured in transillumination experiments. The analysis is based on a lattice random walk formalism which takes into account effects of both positive and negative deviations of the scattering coefficient from that of the bulk. Our simulations indicate the existence of a qualitative difference between the effects of these two types of perturbations. In the case of positive perturbations the time delay is found to be proportional to the square of the size of the inclusion while for negative perturbations the time delay is a linear function of its volume.

1. Introduction

The detection of regions of tissue with abnormal optical parameters is complicated by the fact that tissue is a highly scattering medium. However, the diagnostic value of optical and near-infrared imaging can be enhanced if optical characteristics of the abnormality and surrounding tissue are estimated using different wavelengths. Several theoretical models of photon migration in turbid media have been utilized to quantify optical coefficients as spectroscopic signatures of regions of abnormal tissue embedded in otherwise normal tissue. The most widely used of these models, because of its simplicity, is based on diffusion theory. Such models are either diffusion approximations to a more comprehensive transport equation, or are based on random walk theory (RWT) (Bonner *et al* 1987, Gandjbakhche and Weiss

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1995). These theories can potentially be used to separate the scattering and absorption coefficients internal to the tissue, as estimated from light intensities measured at the tissue interface. Changes in these coefficients can be used to detect an abnormal region in the tissue. In many practically important experimental arrangements the medium can be considered to be an infinite turbid slab bounded by absorbing planes (see, e.g., the arrangement used for optical mammography (Grosenick *et al* 1999)). For simplicity we limit our investigation to this case.

The effect of scattering and absorptive inhomogeneities on photon migration was first considered in the pioneering work of Arridge (1995) using a perturbation analysis (Born approximation) to the diffusion model in the time domain. Similar expressions were obtained later and experimentally verified, using phantoms, by a research group in Quebec (Morin *et al* 2000). Deviations from the perturbation model have been observed for relatively small perturbation amplitudes both for absorptive and scattering inclusions. A more recent study used extensive Monte Carlo simulations of photon migration not only to substantiate the perturbation model for very small inclusions (Carraresi *et al* 2001), but also to estimate the accuracy of the model for a broad range of parameters of an inclusion.

Our previous analysis of absorptive inclusions, based on RWT, demonstrated that an effect of the deviations of absorption coefficients from uniformity on observed intensity contrast cannot be adequately described by a Born approximation perturbation model (Chernomordik *et al* 2000, 2002), for quite small perturbations a dimensionless parameter has been found that determines the accuracy of the perturbation approach. In particular, the contrast amplitude was found not to be proportional to the inclusion volume, except for very small perturbation amplitudes and/or sizes of the anomalous inclusion. It was shown that for relatively large non-localized absorptive inclusions an accurate description of the contrast amplitudes requires an additional correction factor (see also a recent paper by Heffer and Fantini (2002)).

For scattering perturbations we can also expect that effects of non-localization may play an important role in determining the contrast amplitude. They lead to a less sensitive dependence of the contrast on inclusion size than might be expected from a simple perturbation model (Chernomordik *et al* 2000). A RWT analysis of scattering abnormalities shows that photons visiting the region with increased scattering are delayed in reaching the side of the slab opposite to the entrance point (Chernomordik *et al* 2000, Gandjbakhche *et al* 1998). In the framework of RWT, time-resolved contrast functions, used to estimate optical parameters of the abnormal regions, are expected to be proportional to this time delay. The dependence of contrast functions on the size of the inclusion is determined by the relation of the size and the photon time delay due to increased scattering. A limited set of experimental data (Morin *et al* 2000, Painchaud *et al* 1999) supports the conclusion of a RWT analysis that scattering contrast amplitudes are proportional to the cross-section of the inclusion rather than its volume, as predicted by a perturbation analysis based on diffusion theory (Morin *et al* 2000). Until now there have been no simulations of the lattice random walk to substantiate the predicted dependence of the size of the abnormality for a scattering coefficient greater than that of the bulk. There is neither theory nor simulations that relate to negative scattering effects in which the delay is less than that of the bulk. Published data by Morin *et al* (2000) suggest that with negative scattering the contrast is proportional to the inclusion volume.

It should be noted that both cases of positive and negative scattering perturbations are relevant to quantification of optical parameters of tissue abnormalities *in vivo*. Recent analysis of time-resolved optical mammomograms, recorded from breast cancer patients (Grosenick *et al* 2003, 2004) demonstrated that both signs of scattering perturbations can be realized in breast tumours. The range of relative perturbations in scattering coefficients, estimated from a model of diffraction of photon density waves for spherical inclusions, due to tumours is $\mu'_{s,\mathrm{inc}}/\mu'_{s,\mathrm{bulk}} \sim 0.5$–$2.5$ (Grosenick *et al* 2004).

In the present paper we describe a simulation based on a lattice random walk to study the effects of an inclusion in a slab whose scattering properties differ from those of the bulk. The specific experiment to be considered is the transillumination measurement (see, e.g., Chernomordik *et al* (2000), Grosenick *et al* (2003, 2004)) in which a laser beam irradiates a point on one side of a slab and the photon output is measured at the diametrically opposite point. In our investigation the geometry of the tissue is modelled in terms of a simple cubic lattice, which simplifies a simulation study since both the input and detecting optodes are assumed to be points. The aim of our simulations is to find the increment in time, positive or negative, required to reach the detecting optode in the presence of the anomalous inclusion. We also analyse its dependence on some of the geometric features of the inclusion. In this report we assume that the absorption coefficients in the bulk and in the inclusion are equal.

2. Description of the system

In our random walk model the photon is represented by an isotropic nearest-neighbour random walk on a simple cubic lattice as shown in figure 1(a). In the present case the geometry consists of an infinite slab of width L, which may or may not contain an inclusion with scattering properties differing from those of the bulk. An arbitrary point in the slab will be denoted by $\mathbf{r} = (x, y, z)$ where x, y and z are integers. The two faces of the slab are at $z = 0$ and $z = L$. Both interfaces of the slab are assumed to consist only of absorbing points so that random walkers that reach any point on either interface instantly disappear from the system. It should be noted that in a more realistic set-up, boundaries might not be completely absorbing. However, index mismatch, responsible for internal reflection at the boundaries, can be easily incorporated into the theoretical model of photon migration by introducing an extrapolated boundary condition, as has been suggested in the literature (Aronson 1995, Haskell *et al* 1994). The extrapolated boundary condition is defined by requiring zero fluence to occur on a plane parallel to the boundary but shifted outward from the scattering medium. The relationship between this shift, d_e, and reflective properties of the boundary and scattering coefficient of the medium is described in the papers (Aronson 1995, Haskell *et al* 1994). An application of the extrapolated boundary condition for the random walk model has been discussed in our paper (Chernomordik *et al* 1999). The index mismatch is equivalent to a known change of the effective thickness of the slab (Aronson 1995, Haskell *et al* 1994). A photon, or its surrogate, the random walker, is assumed to be initially at the point $\mathbf{r}_0 = (0, 0, 1)$ with the detecting point at $(0, 0, L)$. This definition corresponds to the transillumination experiment (see, e.g., Chernomordik *et al* (2000)). In order to simulate the altered scattering mechanism in the inclusion we add or subtract scattering points to the inclusion as illustrated by the configuration in figure 1(b), in which the density of points within the inclusion is greater than that in the bulk. Similarly, decreased scattering can be simulated by removing points within the inclusion. Note that when points are added in the inclusion, photons can jump back into the bulk at any boundary points, including non-connection lattice points, where they are allowed to jump to the nearest point at the external lattice. In the absence of an inclusion, the random walk takes a step to any one of its six nearest-neighbouring sites with probability $1/6$. At the boundary, we also assume that the total probability for the random walker to leave the inclusion is $1/6$, equal to the probability to jump to one of the 5 neighbouring points inside the inclusion. The parameter to be used to quantify results is the average time for a random walker to reach the detector. The contrast is defined to be the relative difference between the mean transit times with and without the inclusion, which we write as $\Delta\tau = t_{\mathrm{inc}} - t_0$ where t_{inc} is the average time to reach the detecting optode with the anomalous inclusion and t_0 is the average time in the absence of an inclusion.

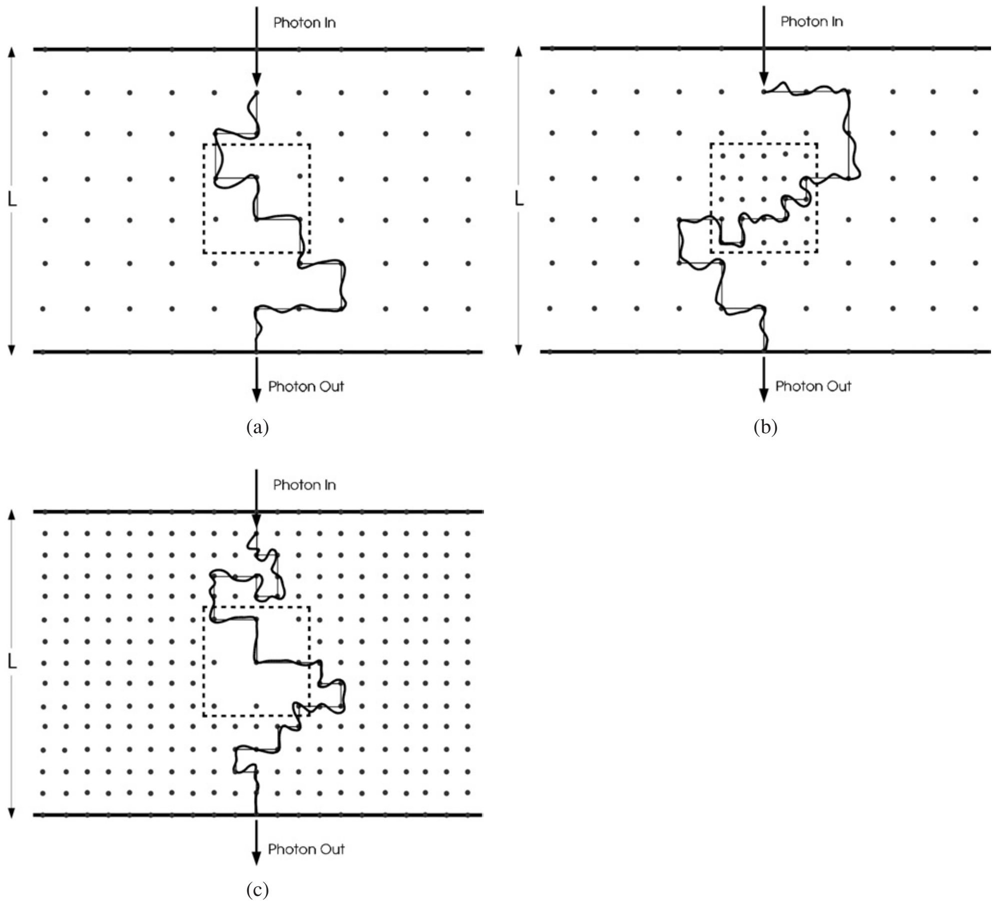

Figure 1. A schematic representation of photon migration in the slab in terms of a random walk on a cubic lattice. The trajectories show photons migrating on their way from the source to detector. The inclusions are shown surrounded by a dashing line: (a) no scattering perturbation, (b) positive scattering perturbation, (c) positive scattering perturbation.

The number of replications in the simulations for each configuration was 2×10^9 random walks in a slab in which the distance between the two faces for the homogeneous lattice was held fixed at $L = 40$. A large number of random walks are required because only a small fraction of the random walkers can be expected to reach the detector. The order of magnitude of the number of detected random walkers which reach the inclusion, at least once, in the present simulations was found to be approximately 10^5. In these simulations the mean times between successive steps were taken to be exponentially distributed random variables. The unit of time can be taken equal to $1/k$ where k is the rate at which steps change directions. This is equivalent to setting $k = 1$. In physical units $k = c\mu'_{s,\mathrm{bulk}}$, $\mu'_{s,\mathrm{bulk}}$ being the transport-corrected scattering coefficient in the bulk of the slab, and c the speed of light in the tissue. The lattice spacing is set equal to unity. Under assumption of a constant distance between successive scattering events this spacing corresponds to the step of length $1/\mu'_{s,\mathrm{bulk}}$ in physical variables. It is worth noting that for a negative exponential distribution of distances between scatterers, usually realized in experiments, with a mean free path equal to $1/\mu'_{s,\mathrm{bulk}}$ the lattice step in physical units proves to be $\sqrt{2}/\mu'_{s,\mathrm{bulk}}$ (Gandjbakhche *et al* 1992).

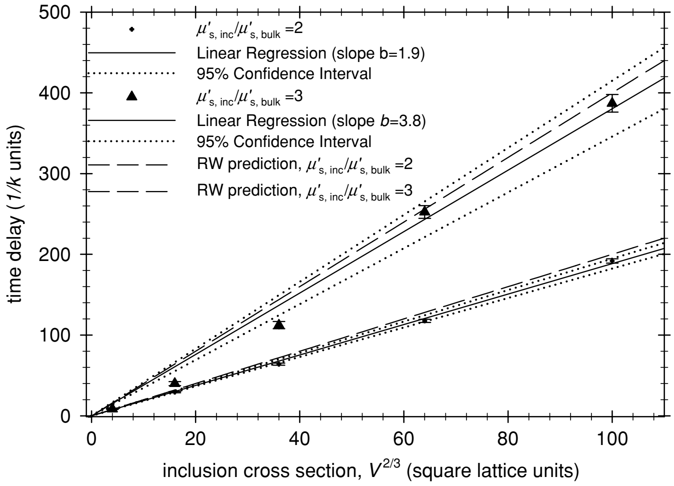

Figure 2. Dependence of the contrast (time delay $\Delta\tau$) on the area of the cubic inclusion (increased scattering) for two ratios of the scattering coefficients inside and outside of the inclusion $\mu'_{s,\text{inc}}/\mu'_{s,\text{bulk}} = 2, 3$.

3. Results

In all of the simulations the centre of the inclusion was set at a point between the interfaces of the slab. Two cases were investigated. In the first, the inclusion was defined by interpolating a single additional lattice point between two of the original lattice points, and in the second, two additional lattice points were interpolated. These intercalations correspond to fixing the ratio of scattering coefficients so that $\mu'_{s,\text{inc}}/\mu'_{s,\text{bulk}} = 2$ and 3 respectively, where $\mu'_{s,\text{inc}}$ is the transport-corrected scattering coefficient in the inclusion.

According to the random walk model (see equation (6) in Gandjbakhche *et al* (1998)), the expected extra time taken by a random walker to migrate a distance $\tilde{d}$ (in lattice units) owing to increased scattering is

$$\Delta\tau = \tilde{d}^2(\tilde{\mu}'_{s,\text{inc}} - 1) = \tilde{d}^2 \frac{\Delta\mu'_s}{\mu'_{s,\text{bulk}}}, \tag{3.1}$$

where $\tilde{\mu}'_{s,\text{inc}} = \frac{\mu'_{s,\text{inc}}}{\mu'_{s,\text{bulk}}}$ is a relative scattering perturbation.

It should be noted that in the usually considered case of negative exponential distribution of scattering lengths, the relationship between the physical size d and dimensionless distance $\tilde{d}$ is, as follows, $\tilde{d} = \frac{\mu'_{s,\text{bulk}}}{\sqrt{2}} d$ (see, e.g., Gandjbakhche *et al* (1998), Gandjbakhche and Weiss (1995)). However, in the present Monte Carlo simulations we assume a constant scattering length in the bulk and a different, but also constant step inside the inclusion. The corresponding lattice step is $1/\mu'_s$. Thus, $\tilde{d} = \mu_{s,\text{bulk}} d$.

Since only half of the random walkers, reaching the other side of the inclusion are actually leaving it, while another half is going back into the inclusion to cross it once more in the opposite direction, the mean extra time $\Delta\tau_{\text{inc}}$ that random walkers (photons) spend inside the inclusion of physical size d, crossing it back and forth before being detected on the slab surface, is equal to

$$\Delta\tau_{\text{inc}} = (\mu'_{s,\text{bulk}} d)^2 \frac{\mu'_{s,\text{inc}}}{\mu'_{s,\text{bulk}}} \sum_{i=0}^{\infty} \left(\frac{1}{2}\right)^i = K(\mu'_{s,\text{bulk}} d)^2 \frac{\Delta\mu'_s}{\mu'_{s,\text{bulk}}}, \tag{3.2}$$

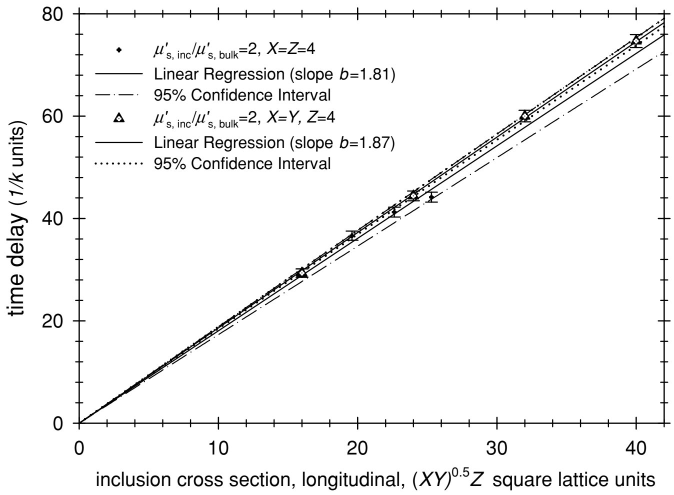

Figure 3. Dependence of the contrast (time delay $\Delta\tau$) on the characteristic area $(XY)^{1/2}Z$ of the parallelepiped inclusion (increased scattering) for different shapes of the parallelepiped, the ratio of the scattering coefficients inside and outside of the inclusion $\mu'_{s,\mathrm{inc}}/\mu'_{s,\mathrm{bulk}} = 2$.

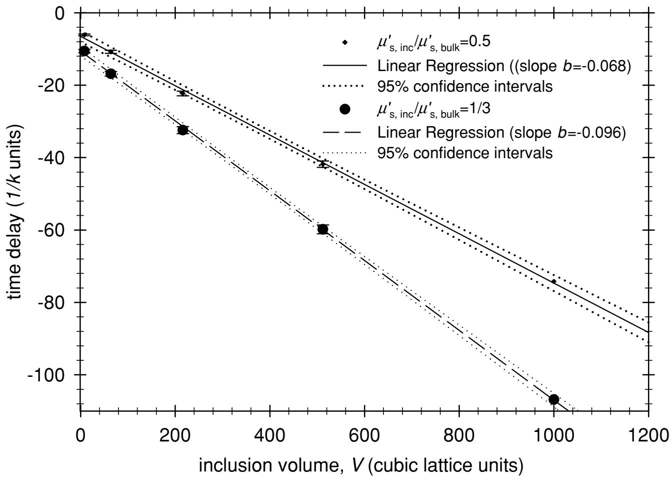

Figure 4. Dependence of the contrast (time delay $\Delta\tau$) on the area of the cubic inclusion (decreased scattering) for two ratios of the scattering coefficients inside and outside of the inclusion $\mu'_{s,\mathrm{inc}}/\mu'_{s,\mathrm{bulk}} = 1/2, 1/3$.

where $K = 2$. Thus, if inclusion sizes are measured in lattice units, and $\tilde{\mu}'_{s,\mathrm{inc}} = \frac{\mu'_{s,\mathrm{inc}}}{\mu'_{s,\mathrm{bulk}}}$ is a relative scattering perturbation, the corresponding time delay for detected photons is

$$\Delta\tau = \Delta\tau_{\mathrm{inc}} = 2\tilde{d}^2\tilde{\mu}'_{s,\mathrm{inc}}. \tag{3.3}$$

When the inclusion is a cube in which additional points are added to the interior of the cube (i.e., a positive deviation) the contrast, or time delay, was found to increase and is seen to be approximately proportional to the area of one side of the cube $\tilde{d}^2$as is evident from the data plotted in figure 2.

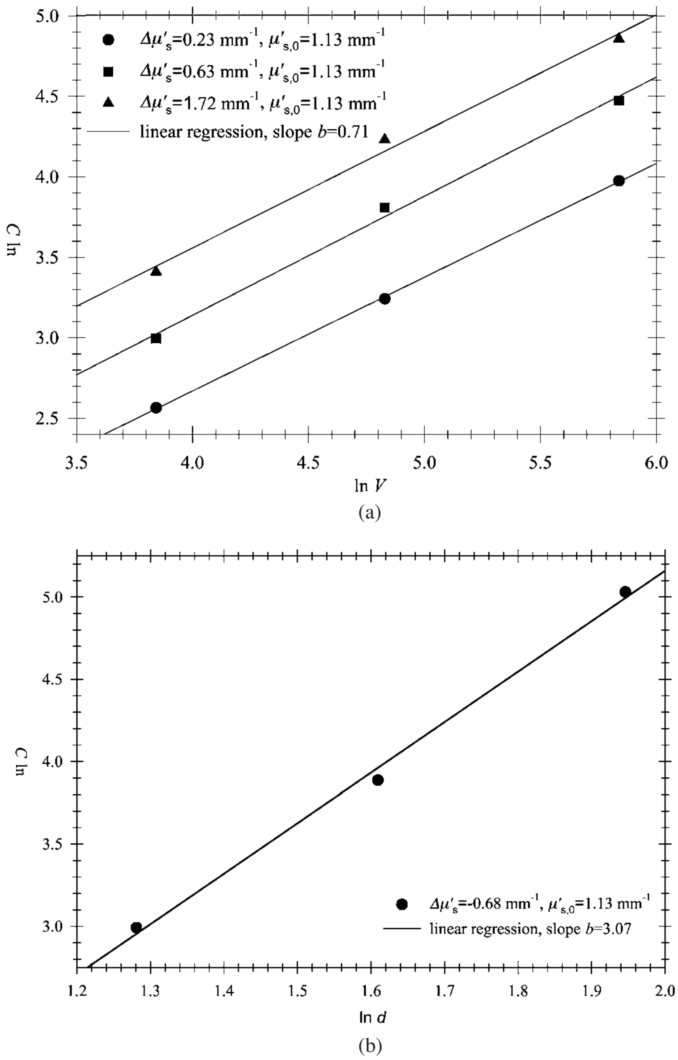

Figure 5. Dependence of contrast amplitude (arbitrary units) on the inclusion volume, based on the experimental data from Morin *et al* (2000) for scattering inclusions, positioned at the midplane of the highly scattering slab ($\mu'_{s,\text{bulk}} = 1.13\,\text{mm}^{-1}$) of thickness $L = 20$ mm that differ only in size ($d = 3.6, 5, 7$ mm). (a) Increased scattering, (b) decreased scattering.

In the case of $\mu'_{s,\text{inc}}/\mu'_{s,\text{bulk}} = 2$, an estimate of the corresponding slope gives the value of $K = 1.9 \pm 0.06$, compared to the theoretical prediction of $K = 2$ as in equation (3.3). For $\mu'_{s,\text{inc}}/\mu'_{s,\text{bulk}} = 3$ the expected value of this slope is $K = 4$, while our simulations give $K = 3.8 \pm 0.36$. The error values given above correspond to 95% confidence intervals.

To further explore the dependence on shape we calculated the contrast when the inclusion of volume V is a parallelepiped with rectangular sides X, Y and Z. When the scattering coefficient is greater than that of the bulk, that is, points were added inside the parallelepiped, we could produce results approximately equivalent to the behaviour found for a cubic inclusion (see figure 3), i.e., the time delay is proportional to a squared characteristic length derived from lengths of sides of the inclusion:

$$\Delta\tau = K(XY)^{1/2}Z \tag{3.4}$$

with an estimated slope of $K \approx 1.8$–1.9, practically independent of the orientation of the parallelepiped as opposed to $K = 1.9$, estimated for the cube.

In contrast, when lattice points are removed from the interior of the inclusion rather than being added to it the behaviour of the time delay of the random walker proved to be qualitatively different: the time delay, being negative, decreases linearly with the volume, as is illustrated by figure 4 (the cubic inclusion case) for two ratios of scattering coefficients, $\mu'_{s,\mathrm{inc}}/\mu'_{s,\mathrm{bulk}} = 1/2$ and $1/3$. The ratio of the slopes, $K_1/K_2 \approx 0.72$, for these two cases substantiates an assumption of the time delay being proportional to the scattering perturbation $\Delta\tilde{\mu}'_s = (\mu'_{s,\mathrm{inc}} - \mu'_{s,\mathrm{bulk}})/\mu'_{s,\mathrm{bulk}}$ (the corresponding ratio is $\Delta\tilde{\mu}'_{s,1}/\Delta\tilde{\mu}'_{s,2} \approx 0.75$), similar to the case of increased scattering coefficients (compare with equation (3.3)).

4. Discussion

The results of simulations described here demonstrate a qualitative difference between the effects induced by positive and negative deviations of the scattering coefficients as described above. In the first case the observed delay in the arrival is proportional to an effective area of the inclusion, as defined in equation (3.4), while in the latter it depends linearly on the inclusion volume. Analytical formulae, describing time delays for negative scattering perturbations, similar to equation (3.4), are not available at the present time.

It is worth noting that a large set of experimental data for scattering perturbations that differed only in size, embedded in a highly scattering slab, was described in the paper (Morin *et al* 2000). Our analysis of these data provides evidence in favour of different scaling properties that depend on whether the scattering coefficient of the inclusion is greater or less than that of the bulk. In particular, we have found that in the first case the contrast amplitudes were proportional to the squared size of inclusions (Chernomordik *et al* 2000), while in the second case these amplitudes depended linearly on the inclusion volumes. These results are illustrated by figures 5(a) and (b), where contrast amplitudes, corresponding to experimental data of Morin *et al* (2000) for scattering inclusions, positioned at the midplane of the highly scattering slab are presented ((a) increased scattering, (b) decreased scattering). These contrast amplitudes were calculated from the results of inclusion characterization (table 1 of Morin *et al* (2000)), comparing measured values of scattering perturbations of three sizes ($d = 3.6$, 5, 7 mm) with values obtained by the authors from the perturbation model (see equations (21), (26), (27b) of the cited paper).

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